C1 January 2007

1. Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find $\frac{dy}{dx}$.

(4)

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2. (a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(1)

(b) Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found.

(3)

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3. Given that $$f(x) = \frac{1}{x}, \quad x \neq 0,$$

(a) sketch the graph of $y = f(x) + 3$ and state the equations of the asymptotes.

(4)

(b) Find the coordinates of the point where $y = f(x) + 3$ crosses a coordinate axis.

(2)

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4. Solve the simultaneous equations

$$y = x - 2,$$

$$y^2 + x^2 = 10.$$

(7)

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5. The equation $2x^2 - 3x - (k + 1) = 0$, where k is a constant, has no real roots.

Find the set of possible values of k.

(4)

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6. (a) Show that $(4 + 3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$, where k is a constant to be found.

(2)

(b) Find $\int (4 + 3\sqrt{x})^2 \, dx$.

(3)

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7. The curve C has equation $y = \mathrm{f}(x)$, $x \neq 0$, and the point P (2, 1) lies on C. Given that

$$\mathrm{f}'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find f(x).

(5)

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

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8. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $\quad x > 0$.

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

(b) Show that the point P (4, 8) lies on C.

(1)

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form.

(3)

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9. Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

Row 1 

Row 2 

Row 3 

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

(a) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the nth row.

(3)

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

(b) Find the total number of sticks Ann uses in making these 10 rows.

(3)

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k+1)$th row,

(c) show that k satisfies $(3k-100)(k+35)<0$.

(4)

(d) Find the value of k.

(2)

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10. (a) On the same axes sketch the graphs of the curves with equations

(i) $y=x^2(x-2)$,

(3)

(ii) $y=x(6-x)$,

(3)

and indicate on your sketches the coordinates of all the points where the curves cross the x-axis.

(b) Use algebra to find the coordinates of the points where the graphs intersect.

(7)

TOTAL FOR PAPER: 75 MARKS

January 2007
6663 Core Mathematics C1
Mark Scheme

Question Number	Scheme	Mark
1.	$4x^3 \to kx^2$ or $2x^{\frac{1}{2}} \to kx^{-\frac{1}{2}}$ (k a non-zero constant)	M1
	$12x^2, + x^{-\frac{1}{2}}\ldots\ldots,$ $(-1 \to 0)$	A1, A1, B1 (4)
		4
	Accept equivalent alternatives to $x^{-\frac{1}{2}}$, e.g. $\frac{1}{x^{1/2}}$, $\frac{1}{\sqrt{x}}$, $x^{-0.5}$. M1: $4x^3$ 'differentiated' to give kx^2, or… $2x^{\frac{1}{2}}$ 'differentiated' to give $kx^{-\frac{1}{2}}$ (but not for just $-1 \to 0$). 1st A1: $12x^2$ (Do not allow just $3 \times 4x^2$) 2nd A1: $x^{-\frac{1}{2}}$ or equivalent. (Do not allow just $\frac{1}{2} \times 2x^{-\frac{1}{2}}$, but allow $1x^{-\frac{1}{2}}$ or $\frac{2}{2}x^{-\frac{1}{2}}$). B1: -1 differentiated to give zero (or 'disappearing'). Can be given provided that at least one of the other terms has been changed. Adding an extra term, e.g. + C, is B0.	

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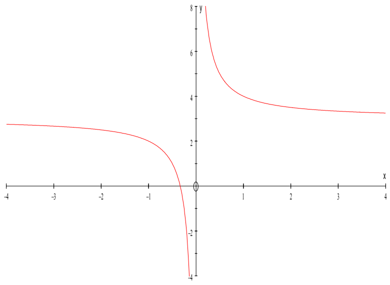
Question Number	Scheme		Marks	
2.	(a) $6\sqrt{3}$	$(a = 6)$	B1	(1)
	(b) Expanding $(2-\sqrt{3})^2$ to get 3 or 4 separate terms		M1	
	$7,\ -4\sqrt{3}$	$(b = 7,\ c = -4)$	A1, A1	(3)
				4
	(a) $\pm 6\sqrt{3}$ also scores B1. (b) M1: The 3 or 4 terms may be wrong. 1st A1 for 7, 2nd A1 for $-4\sqrt{3}$. Correct answer $7-4\sqrt{3}$ with no working scores all 3 marks. $7+4\sqrt{3}$ with or without working scores M1 A1 A0. Other wrong answers with no working score no marks.			

Question Number	Scheme		Marks	
3.	(a) 	Shape of f(x)	B1	
		Moved up ↑	M1	
		Asymptotes: $y = 3$	B1	
		$x = 0$ (Allow "y-axis")	B1	(4)
		($y \neq 3$ is B0, $x \neq 0$ is B0).		
	(b) $\frac{1}{x} + 3 = 0$	No variations accepted.	M1	
	$x = -\frac{1}{3}$ (or $-0.33\ldots$)	Decimal answer requires at least 2 d.p.	A1	(2)
				6

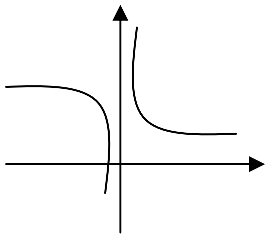
(a) B1: Shape requires both branches and no obvious "overlap" with the asymptotes (see below), but otherwise this mark is awarded generously. The curve may, e.g., bend away from the asymptote a little at the end. Sufficient curve must be seen to suggest the asymptotic behaviour, both horizontal and vertical.

M1: Evidence of an upward translation parallel to the y-axis. The shape of the graph can be wrong, but the complete graph (both branches if they have 2 branches) must be translated upwards. This mark can be awarded generously by implication where the graph drawn is an upward translation of another standard curve (but not a straight line).

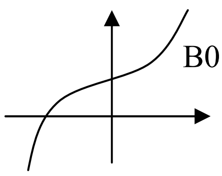
The B marks for asymptote equations are independent of the graph.

Ignore extra asymptote equations, if seen.

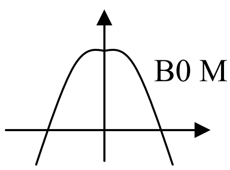
(b) Correct answer with no working scores both marks.
The answer may be seen on the sketch in part (a).
Ignore any attempts to find an intersection with the y-axis.

e.g. 

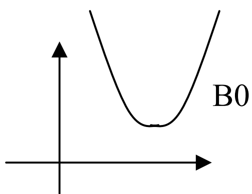
(a) This scores B0 (clear overlap with horiz. asymp.)
M1 (Upward translation… bod that both branches have been translated).



B0 M1



B0 M1



B0 M0

No marks unless the original curve is seen, to show upward translation.

Question Number	Scheme	Marks
4.	$(x-2)^2 = x^2 - 4x + 4$ or $(y+2)^2 = y^2 + 4y + 4$ M: 3 or 4 terms	M1
	$(x-2)^2 + x^2 = 10$ or $y^2 + (y+2)^2 = 10$ M: Substitute	M1
	$2x^2 - 4x - 6 = 0$ or $2y^2 + 4y - 6 = 0$ Correct 3 terms	A1
	$(x-3)(x+1) = 0, \quad x = ...$ or $(y+3)(y-1) = 0, \quad y = ...$ (The above factorisations may also appear as $(2x-6)(x+1)$ or equivalent).	M1
	$x = 3 \quad x = -1$ or $y = -3 \quad y = 1$	A1
	$y = 1 \quad y = -3$ or $x = -1 \quad x = 3$	M1 A1 (7)
	(Allow equivalent fractions such as: $x = \frac{6}{2}$ for $x = 3$).	**7**
	1st M: 'Squaring a bracket', needs 3 or 4 terms, one of which must be an x^2 or y^2 term. 2nd M: Substituting to get an equation in one variable (awarded generously). 1st A: Accept equivalent forms, e.g. $2x^2 - 4x = 6$. 3rd M: Attempting to solve a 3-term quadratic, to get 2 solutions. 4th M: Attempting at least one y value (or x value). If y solutions are given as x values, or vice-versa, penalise at the end, so that it is possible to score M1 M1A1 M1 A1 M0 A0. Strict "pairing of values" at the end is <u>not</u> required. <u>"Non-algebraic" solutions:</u> No working, and only one correct solution pair found (e.g. $x = 3$, $y = 1$): M0 M0 A0 M0 A0 M1 A0 No working, and both correct solution pairs found, but not demonstrated: M0 M0 A0 M1 A1 M1 A1 Both correct solution pairs found, and demonstrated, perhaps in a table of values: Full marks <u>Squaring individual terms:</u> e.g. $y^2 = x^2 + 4$ M0 $x^2 + 4 + x^2 = 10$ M1 A0 (Eqn. in one variable) $x = \sqrt{3}$ M0 A0 (Not solving 3-term quad.) $y^2 = x^2 + 4 = 7 \quad y = \sqrt{7}$ M1 A0 (Attempting one y value)	

Question Number	Scheme	Marks	
5.	Use of $b^2 - 4ac$, perhaps implicit (e.g. in quadratic formula)	M1	
	$(-3)^2 - 4 \times 2 \times -(k+1) < 0$ $\quad (9 + 8(k+1) < 0)$	A1	
	$8k < -17$ (Manipulate to get $pk < q$, or $pk > q$, or $pk = q$)	M1	
	$k < -\frac{17}{8}$ $\quad \left(\text{Or equiv}: k < -2\frac{1}{8} \text{ or } k < -2.125\right)$	A1cso	(4)
			4

1st M: Could also be, for example, comparing or equating b^2 and $4ac$.
Must be considering the given quadratic equation.
There must not be x terms in the expression, but there must be a k term.

1st A: Correct expression (need not be simplified) and correct inequality sign.
Allow also $-3^2 - 4 \times 2 \times -(k+1) < 0$.

2nd M: Condone sign or bracketing mistakes in manipulation.
Not dependent on 1st M, but should not be given for irrelevant work.
M0 M1 could be scored:
e.g. where $b^2 + 4ac$ is used instead of $b^2 - 4ac$.

Special cases:

1. Where there are x terms in the discriminant expression, but then division by x^2 gives an inequality/equation in k. (This could score M0 A0 M1 A1).
2. Use of $\leq$ instead of $<$ loses one A mark only, at first occurrence, so an otherwise correct solution leading to $k \leq -\frac{17}{8}$ scores M1 A0 M1 A1.

N.B. Use of $b = 3$ instead of $b = -3$ implies no A marks.

Question Number	Scheme	Marks
6.	(a) $(4+3\sqrt{x})(4+3\sqrt{x})$ seen, or a numerical value of k seen, $(k \neq 0)$. (The k value need not be explicitly stated… see below). $16+24\sqrt{x}+9x$, or $k=24$ (b) $16 \to cx$ or $kx^{1/2} \to cx^{3/2}$ or $9x \to cx^2$ $\int(16+24\sqrt{x}+9x)\mathrm{d}x = 16x + \frac{9x^2}{2} + C, \ +16x^{3/2}$	M1 A1cso (2) M1 A1, A1ft (3) **5**
	(a) e.g. $(4+3\sqrt{x})(4+3\sqrt{x})$ alone scores M1 A0, (but <u>not</u> $(4+3\sqrt{x})^2$ alone). e.g $16+12\sqrt{x}+9x$ scores M1 A0. $k=24$ or $16+24\sqrt{x}+9x$, with no further evidence, scores full marks M1 A1. Correct solution only (cso): any wrong working seen loses the A mark. (b) A1: $16x+\frac{9x^2}{2}+C$. Allow 4.5 or $4\frac{1}{2}$ as equivalent to $\frac{9}{2}$. A1ft: $\frac{2k}{3}x^{3/2}$ (candidate's value of k, or general k). For this final mark, allow for example $\frac{48}{3}$ as equivalent to 16, but do <u>not</u> allow unsimplified "double fractions" such as $\frac{24}{\left(3/2\right)}$, and do <u>not</u> allow unsimplified "products" such as $\frac{2}{3}\times 24$. A single term is required, e.g. $8x^{3/2}+8x^{3/2}$ is not enough. An otherwise correct solution with, say, C missing, followed by an incorrect solution including + C can be awarded full marks (isw, but allowing the C to appear at any stage).	

Question Number	Scheme		Marks	
7.	(a) $3x^2 \to cx^3$ or $-6 \to cx$ or $-8x^{-2} \to cx^{-1}$		M1	
	$f(x) = \frac{3x^3}{3} - 6x - \frac{8x^{-1}}{-1}$ $(+C)$ $\left(x^3 - 6x + \frac{8}{x}\right)$		A1 A1	
	Substitute $x = 2$ and $y = 1$ into a 'changed function' to form an equation in C.		M1	
	$1 = 8 - 12 + 4 + C$ $C = 1$		A1cso	(5)
	(b) $3 \times 2^2 - 6 - \frac{8}{2^2}$		M1	
	$= 4$		A1	
	Eqn. of tangent: $y - 1 = 4(x - 2)$		M1	
	$y = 4x - 7$	(Must be in this form)	A1	(4)
				9

(a) First 2 A marks: + C is not required, and coefficients need not be simplified, but powers must be simplified.

All 3 terms correct: A1 A1
Two terms correct: A1 A0
Only one term correct: A0 A0

Allow the M1 A1 for finding C to be scored either in part (a) or in part (b).

(b) 1st M: Substituting $x = 2$ into $3x^2 - 6 - \frac{8}{x^2}$ (must be this function).

2nd M: Awarded generously for attempting the equation of a straight line through (2, 1) or (1, 2) with any value of m, however found.

2nd M: Alternative is to use (2, 1) or (1, 2) in $y = mx + c$ to find a value for c.

If calculation for the gradient value is seen in part (a), it must be used in part (b) to score the first M1 A1 in (b).

Using (1, 2) instead of (2, 1): Loses the 2nd method mark in (a).
Gains the 2nd method mark in (b).

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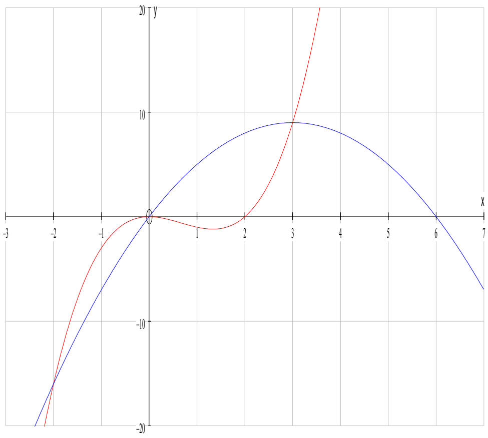
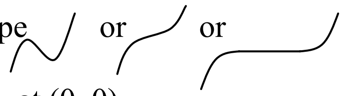
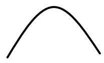
Question Number	Scheme	Marks	
8.	(a) $4x \to k$ or $3x^{3/2} \to kx^{1/2}$ or $-2x^2 \to kx$	M1	
	$\frac{dy}{dx} = 4 + \frac{9}{2}x^{1/2} - 4x$	A1 A1	(3)
	(b) For $x = 4$, $y = (4\times 4) + (3 \times 4\sqrt{4}) - (2 \times 16) = 16 + 24 - 32 = 8$ (*)	B1	(1)
	(c) $\frac{dy}{dx} = 4 + 9 - 16 = -3$ M: Evaluate their $\frac{dy}{dx}$ at $x = 4$	M1	
	Gradient of normal = $\frac{1}{3}$	A1ft	
	Equation of normal: $y - 8 = \frac{1}{3}(x - 4)$, $3y = x + 20$ (*)	M1, A1	(4)
	(d) $y = 0$: $x = ...$ (-20) and use $(x_2 - x_1)^2 + (y_2 - y_1)^2$	M1	
	$PQ = \sqrt{24^2 + 8^2}$ or $PQ^2 = 24^2 + 8^2$ Follow through from (k, 0)	A1ft	
	May also be scored with $(-24)^2$ and $(-8)^2$.		
	$= 8\sqrt{10}$	A1	(3)
			11
	(a) For the 2 A marks coefficients need <u>not</u> be simplified, but powers must be simplified. For example, $\frac{3}{2} \times 3x^{1/2}$ is acceptable. All 3 terms correct: A1 A1 Two terms correct: A1 A0 Only one term correct: A0 A0 (b) There must be some evidence of the "24" value. (c) In this part, beware 'working backwards' from the given answer. A1ft: Follow through is just from the candidate's <u>value</u> of $\frac{dy}{dx}$. 2nd M: Is not given if an m value appears "from nowhere". 2nd M: Must be an attempt at a <u>normal</u> equation, not a tangent. 2nd M: Alternative is to use (4, 8) in $y = mx + c$ to <u>find a value</u> for c. (d) M: Using the normal equation to attempt coordinates of Q, (even if using $x = 0$ instead of $y = 0$), <u>and</u> using Pythagoras to attempt PQ or PQ^2. Follow through from (k, 0), but <u>not</u> from (0, k)… A common wrong answer is to use $x = 0$ to give $\frac{20}{3}$. This scores M1 A0 A0. For final answer, accept other simplifications of $\sqrt{640}$, e.g. $2\sqrt{160}$ or $4\sqrt{40}$.		

Question Number	Scheme	Marks
9.	(a) Recognising arithmetic series with first term 4 and common difference 3. (If not scored here, this mark may be given if seen elsewhere in the solution). $a+(n-1)d = 4+3(n-1) \quad (=3n+1)$	B1 M1 A1 (3)
	(b) $S_n = \frac{n}{2}\{2a+(n-1)d\} = \frac{10}{2}\{8+(10-1)\times 3\}, \quad =175,$	M1 A1, A1 (3)
	(c) $S_k < 1750: \ \frac{k}{2}\{8+3(k-1)\} < 1750 \ \left(\text{or} \ \ S_{k+1} > 1750: \ \frac{k+1}{2}\{8+3k\} > 1750\right)$	M1
	$3k^2+5k-3500<0 \ \left(\text{or} \ 3k^2+11k-3492>0\right)$ (Allow equivalent 3-term versions such as $3k^2+5k=3500$).	M1 A1
	$(3k-100)(k+35)<0$ Requires use of correct inequality throughout.(*)	A1cso (4)
	(d) $\frac{100}{3}$ or equiv. <u>seen</u> $\left(\text{or} \ \frac{97}{3}\right)$, $k = 33$ (and no other values)	M1, A1 (2)
		12
	(a) B1: Usually identified by $a = 4$ and $d = 3$. M1: Attempted use of term formula for arithmetic series, or… answer in the form ($3n$ + constant), where the constant is a non-zero value. Answer for (a) does not require simplification, and a correct answer without working scores all 3 marks. (b) M1: Use of correct sum formula with $n = 9$, 10 or 11. A1: Correct, perhaps unsimplified, numerical version. A1: 175 <u>Alternative</u>: (Listing and summing terms). M1: Summing 9, 10 or 11 terms. (At least 1st, 2nd and last terms must be seen). A1: Correct terms (perhaps implied by last term 31). A1: 175 <u>Alternative</u>: (Listing all sums) M1: Listing 9, 10 or 11 sums. (At least 4, 7, ….., "*last*"). A1: Correct sums, correct finishing value 175. A1: 175 <u>Alternative</u>: (Using last term). M1: Using $S_n = \frac{n}{2}(a+l)$ with T_9, T_{10} or T_{11} as the last term. A1: Correct numerical version $\frac{10}{2}(4+31)$. A1: 175 Correct answer with <u>no</u> working scores 1 mark: 1,0,0. (c) For the first 3 marks, allow <u>any inequality sign</u>, or <u>equals.</u> 1st M: Use of correct sum formula to form inequality or equation in k, with the 1750. 2nd M: (Dependent on 1st M). Form 3-term quadratic in k. 1st A: Correct 3 terms. Allow credit for part (c) if valid work is seen in part (d). (d) Allow both marks for $k = 33$ seen without working. Working for part (d) must be seen in part (d), not part (c).	

Question Number	Scheme	Marks	
10.	(a)  (i) Shape 	B1	
	Max. at (0, 0).	B1	
	(2, 0), (or 2 shown on x-axis).	B1	(3)
	(ii) Shape 	B1	
	(It need not go below x-axis) Through origin.	B1	
	(6, 0), (or 6 shown on x-axis).	B1	(3)
	(b) $x^2(x-2)=x(6-x)$	M1	
	$x^3-x^2-6x=0$ Expand to form 3-term cubic (or 3-term quadratic if divided by x), with all terms on one side. The "= 0" may be implied.	M1	
	$x(x-3)(x+2)=0$ $x=\ldots$ Factor x (or divide by x), and solve quadratic.	M1	
	$x=3$ and $x=-2$	A1	
	$x=-2$: $y=-16$ Attempt y value for a non-zero x value by substituting back into $x^2(x-2)$ or $x(6-x)$.	M1	
	$x=3$: $y=9$ Both y values are needed for A1.	A1	
	$(-2,-16)$ and $(3,9)$		
	(0, 0) This can just be written down. Ignore any 'method' shown. (But must be seen in part (b)).	B1	(7)
			13
	(a) (i) For the third 'shape' shown above, where a section of the graph coincides with the x-axis, the B1 for (2, 0) can still be awarded if the 2 is shown on the x-axis. For the final B1 in (i), and similarly for (6, 0) in (ii): There must be a sketch. If, for example (2, 0) is written separately from the sketch, the sketch must not clearly contradict this. If (0, 2) instead of (2, 0) is shown on the sketch, allow the mark. Ignore extra intersections with the x-axis. (ii) 2nd B is dependent on 1st B. Separate sketches can score all marks. (b) Note the dependence of the first three M marks. A common wrong solution is (-2, 0), (3, 0), (0, 0), which scores M0 A0 B1 as the last 3 marks. A solution using no algebra (e.g. trial and error), can score up to 3 marks: M0 M0 M0 A0 M1 A1 B1. (The final A1 requires both y values). Also, if the cubic is found but not solved algebraically, up to 5 marks: M1 M1 M0 A0 M1 A1 B1. (The final A1 requires both y values).		